An Open-Source Tool for Accurate Topography Implementation within Finite-Difference Wave Solvers

1. Motivation
- Seismic imaging/modeling scenarios such as FWI and migration, along with lithospheric tomography may require complex topography to be accounted for within the numerical scheme:
  - Capturing topographic effects in wave models for FWI
  - Accounting for surface-reflected phases in lithospheric tomography
- Modelling topographic scattering of seismic waves from earthquake events
- In such scenarios, failure to account for topography can lead to degraded image quality, or unrealistic wave behaviour
- Topographic scattering effects require careful representation of the free surface if seismic code is to be accurately modelled
- Surface represents a sharp, irregular discontinuity which is difficult to include in wave propagators based on structured grids, such as finite-difference (FD) solvers
- FD is commonly used in seismic applications as it is conceptually simple, relatively computationally cheap, and has a suite of known optimizations
- Naïve ‘vacuum-layer’ approaches, whilst straightforward have poor stability characteristics and generate spurious scattering artifacts
- We wish to accurately represent complex topography whilst retaining the advantages of structured grids

2. Immersed Boundaries
- Used to impose boundary conditions on a variety of surfaces of arbitrary shape within FD schemes, without geometric transformations
  - Pioneered in fluid-flow simulations
  - Artificial field values are calculated at grid nodes above the surface
  - These values are estimated by constructing extrapolating functions fitted using a combination of interior point values and free-surface boundary conditions
  - As the boundary point where the function is fitted does not need to be coincident with a grid node, boundary conditions can be imposed off-grid

3. Topography Representation
- Boundary surface represented as a signed distance function (SDF) discretized onto FD grid
- SDF generated using the VTK toolkit from a 3D mesh
- Properties of SDF allow sectioning of interior from exterior nodes and straightforward calculation of boundary positions

4. Extrapolation Construction (contd.)
- Lagrange polynomials are truncated to fit function values at interior stencil points and specified boundary conditions independent 1D extrapolation per coordinate direction automatically generated using symbolic computation
- Exterior values given by these polynomials are substituted into FD stencils, removing exterior nodes
- This results in variable stencil coefficients in the boundary-adjacent region
- Removes need for ghost grid

5. Topographic Scattering Model
- Forward model based on 1st-order formulation of the acoustic wave equation to demonstrate modelling of topographic effects
- Zero pressure imposed at the free-surface, in turn implying zero even pressure derivatives and odd velocity derivatives
- Discretization in 4th-order accurate in space and 2nd-order accurate in time
  - 10.8km x 10.8km x 5.4km FD grid with 50m grid spacing

6. Accuracy Comparison (contd.)
- Immersed boundary accuracy compared to a naïve vacuum-layers approach for modelling reflections from a simple hill

7. Devito and Devitoboundary
- Devito is an open-source domain-specific-language (DSL) and compiler embedded in Python
- Allows FD codes to be specified in a high-level formalism based on SymPy
- Generates production-grade C code with nested parallelism and a range of optimizations across multiple architectures
- Can be seamlessly compiled from laptop to HPC
- Devitoboundary is an open-source add-on for Devito which aims to provide a high-level interface for including immersed boundaries in seismic applications
- Immersed boundary is encapsulated in a handful of high-level objects
- Integrates with Devito’s custom coefficients functionality

8. Future Work
We are currently working on an improved extrapolation strategy for vector boundary conditions
We hope to achieve greater accuracy and stability for the first-order acoustic wave equation alongside extension to the elastic wave equation