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DEVITO

An Open-Source Tool for Accurate Topography Implementation within Finite-Difference Wave Solvers

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1. Motivation

- **Seismic imaging/modelling scenarios such as FWI and migration, along with lithospheric tomography[3] may require complex topography to be accounted for within the numerical scheme:**
  - Capturing topographic effects in wave models for FWI
  - Accounting for surface-reflected phases in lithospheric tomography[3]
  - Modelling topographic scattering of seismic waves from earthquake events
- In such scenarios, failure to account for topography can lead to degraded image quality, or unrealistic wave behaviour
- Topographic scattering effects require careful representation of the free surface if seismic coda is to be accurately modelled[4]
- Surface represents a sharp, irregular discontinuity which is difficult to include in wave propagators based on structured grids, such as finite-difference (FD) solvers[1][5][6][7]
- FD is commonly used in seismic applications as it is conceptually simple, relatively computationally cheap, and have a suite of known optimizations[8]
- Naïve 'vacuum-layer' approaches, whilst straightforward have poor stability characteristics and generate spurious scattering artifacts[2]
- We wish to accurately represent complex topography whilst retaining the advantages of structured grids

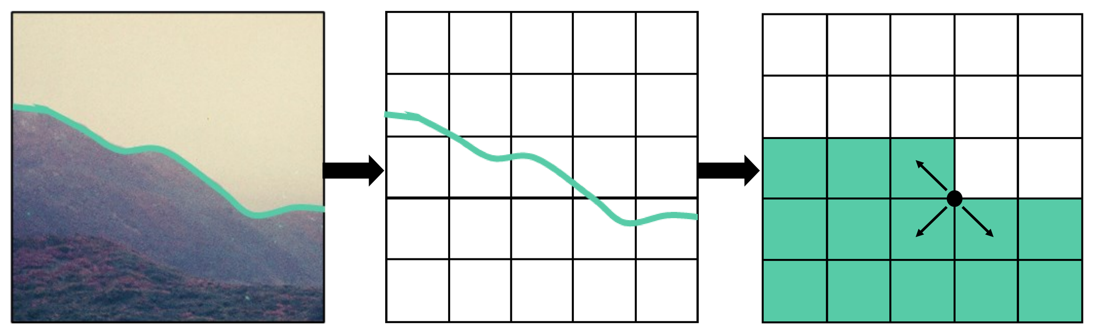

Figure 1: Misrepresentation of topography due to approximation to a regular grid. The smooth topography becomes artificially blocky, produding spurious scatterers, as shown in the third subfigure, where the gentle slope becomes stepped. This staircasing produces both 1st and 2nd-order error[1] due to boundary mislocation and material contrast, alongside spurious scattering[2].

2. Immersed Boundaries

- **Used to impose boundary conditions on smooth surfaces of arbitrary shape within FD schemes, without geometric transformations**
- Pioneered in fluid-flow simulations[9]
- Artificial field values are calculated at grid nodes above the surface
- These values are estimated by constructing extrapolating functions fitted using a combination of interior point values and free-surface boundary conditions
- As the boundary point where the function is fitted does not need to be coincident with a grid node, boundary conditions can be imposed off-grid

3. Topography Representation

- **Boundary surface represented as a signed distance function (SDF) discretized onto FD grid**
- SDF generated using the VTK toolkit from a 3D mesh
- Properties of SDF allow sectioning of interior from exterior nodes and straightforward calculation of boundary positions

4. Extrapolation Construction

- **Lagrange polynomials of equal order to the FD scheme fitted to function values at interior stencil points and specified boundary conditions**
- Independent 1D extrapolation per coordinate direction automatically generated using symbolic computation
- Exterior values given by these polynomials are substituted into FD stencils, removing exterior nodes
- This results in variable stencil coefficients in the boundary-adjacent region.
- Removes need for ghost grid

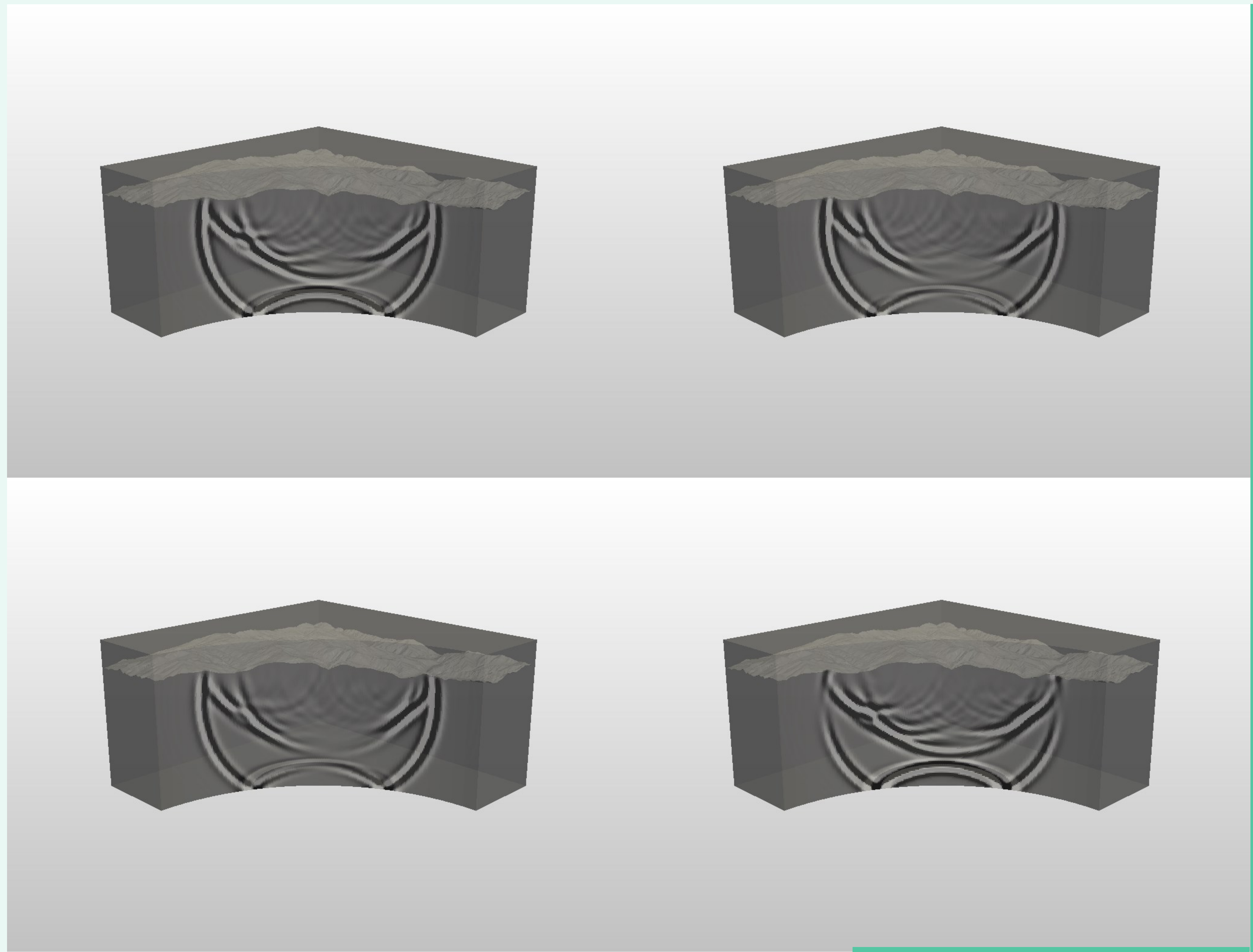

Figure 2: A cutaway render at t=4s of the wavefields for the topographic scattering model. Counter-clockwise from top left, the renders show pressure, followed by particle velocities in the x, y, and z directions respectively. The model run demonstrated expected scattering behaviour with only minor distortion of the outgoing wavefront, but numerous chaotic reflections following it.

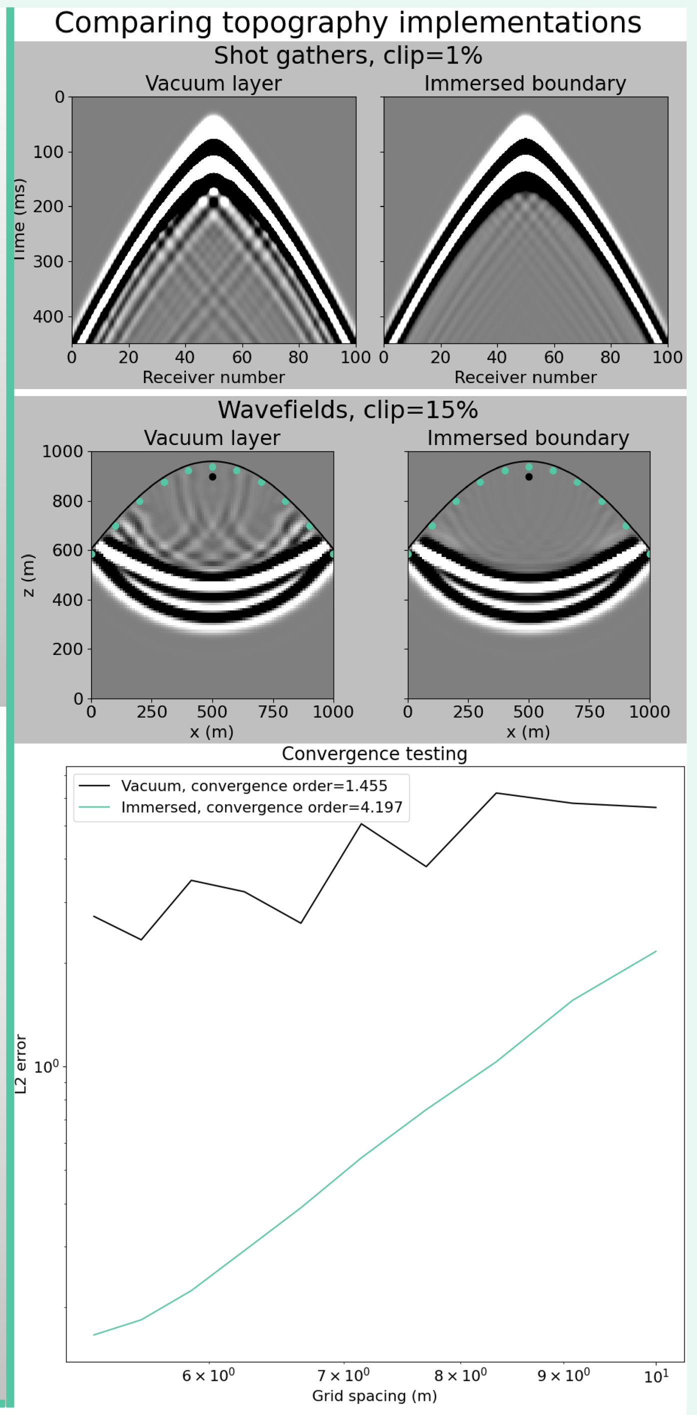

Figure 4: A comparison between vacuum layer and immersed boundary implementations. Black and teal dots are sources and decimated receivers respectively. The vacuum layer exhibits strong scattering artifacts following the reflected wave, which are suppressed in the immersed boundary model, resulting in much cleaner reflected geometry. Convergence testing demonstrates the superior convergence for the immersed boundary approach.

4. Extrapolation Construction (contd.)

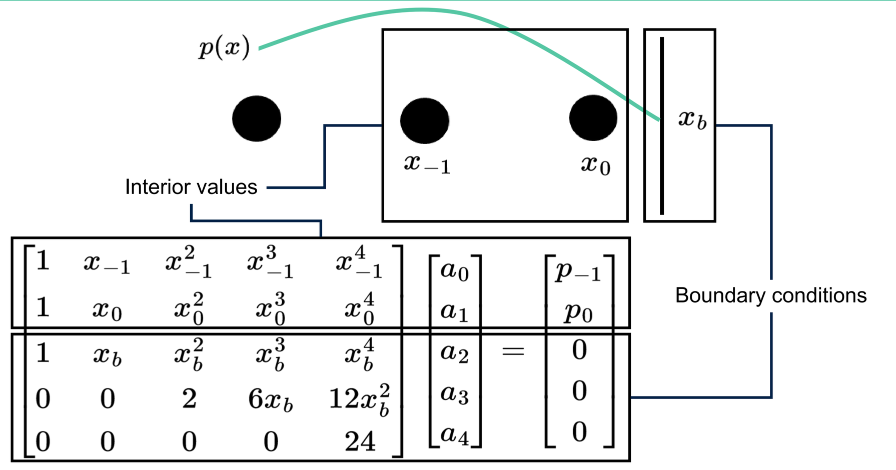

Figure 3: The linear system formed in order to calculate the extrapolation polynomial coefficients, showing where each equation originates. By solving this system for a given set of boundary conditions, polynomial coefficients as functions of interior values and boundary position are obtained. These can then be used to extrapolate the function beyond x_b, to obtain exterior stencil values.

5. Topographic Scattering Model

- **Forward model based on 1st-order formulation of the acoustic wave equation to demonstrate modelling of topographic effects**
- Zero pressure imposed at the free-surface, in turn implying zero even pressure derivatives and odd velocity derivatives.
- Discretization is 4th-order accurate in space and 2nd-order accurate in time
- 10.8km x 10.8km x 5.4km FD grid with 50m grid spacing

5. Topographic Scattering Model (contd.)

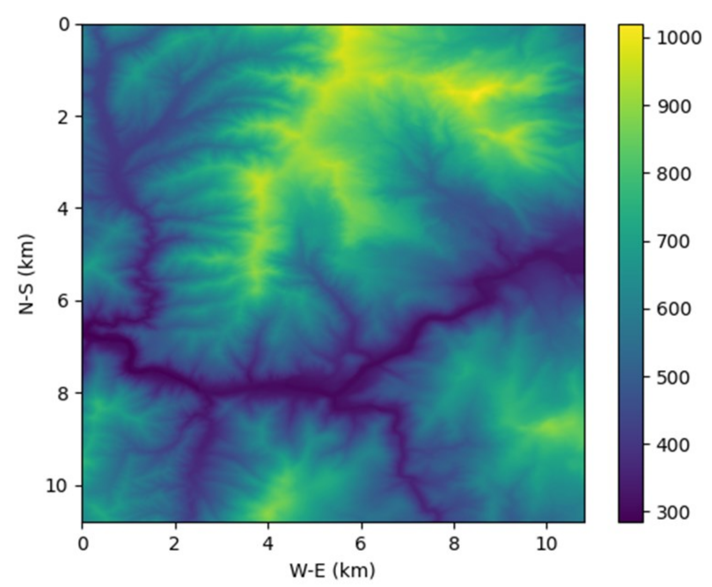

Figure 5: Surface geometry used in the scattering model, taken from 1 arcsecond SRTM DEM of a mountainous area near Umpqua National Forest, Oregon. Scalebar shows elevation in meters.

- Central ricker source positioned 500m below sea level, injected into pressure field
- Homogenous material properties, so all scattering is a product of the boundary treatment

6. Accuracy Comparison

- **Immersed boundary accuracy compared to a naïve vacuum-layer approach for modelling reflections from a simple hill**
- 2nd-order acoustic wave propagator with homogenous material properties
- Discretization is 4th-order accurate in space and 2nd-order accurate in time
- Figure 4 shows both wavefields and shot gathers implemented using the two methods.
- Results of convergence testing against an 8th-order immersed-boundary run on twofold-refined grids are included in figure 4

6. Accuracy Comparison (contd.)

- The immersed-boundary method achieved the specified 4th-order convergence
- Vacuum methods of equivalent order exhibited an order of magnitude more error, and inconsistent convergence with a sub-2nd-order trend

7. Devito and Devitoboundary

```python
from devito import *

grid = grid(shape=(nx, ny), extent=(Lx, Ly))

u = TimeFunction(name='u', grid=grid, space_order=2)
u.data[0, :] = initial_data[:]

eq = Eq(u.dt, a*u.laplace)
stencil = solve(eq, u.forward)

op = Operator(Eq(u.forward, stencil))
op(t=timesteps, dt=dt)
```

Listing 1: A simple 2D diffusion operator in Devito. The symbolic specification lends itself to ease of use, code readability, and accelerated workflow. The FD operator itself is encapsulated in the Operator object.

- **The wavesolvers demonstrated here were implemented using Devito with immersed-boundary treatments generated by Devitoboundary**
- Devito is an open-source domain-specific-language (DSL) and compiler embedded in Python[8]
  - Allows FD codes to be specified in a high-level form based on SymPy
  - Generates production-grade C code with nested parallelism and a range of optimizations across multiple architectures
  - Can be straightforwardly scaled from laptop to HPC
- Devitoboundary is an open-source add-on for Devito which aims to provide a high-level interface for including immersed boundaries in seismic applications
  - Immersed boundary is encapsulated in a handful of high-level objects
  - Integrates with Devito's custom coefficients functionality

8. Future Work

- **We are currently working on an improved extrapolation strategy for vector boundary conditions**
- We hope to achieve greater accuracy and stability for the first-order acoustic wave equation alongside extension to the elastic wave equation

10. Contact and Resources

Email- edward.caunt15@imperial.ac.uk (EdC on Devito Slack)

Website - www.devitoproject.org/

Slack - devitocodes.slack.com

Devito Github - github.com/devitocodes/devito

Devitoboudnary Github - github.com/devitocodes/devitoboundary

11. References

[1] Mulder, W. A. (2017) 'A simple finite-difference scheme for handling topography with the second-order wave equation', Geophysics, 82(3), pp. 111–120.
[2] Hu, W. (2016). An improved immersed boundary finite-difference method for seismic wave propagation modeling with arbitrary surface topography. Geophysics, 81(6), 311–322.
[3] Monteiller, V., Komatitsch, D., & Wang, Y. (2015). Three-dimensional full waveform inversion of short-period teleseismic wavefields based upon the SEM-DSM hybrid method. Geophysical Journal International, 202, 811–827.
[4] Takemura, S., Furumura, T., & Maeda, T. (2015). Scattering of high-frequency seismic waves caused by irregular surface topography and small-scale velocity inhomogeneity. Geophysical Journal International, 201, 459–474.
[5] Zeng, C., Xia, J., Miller, R. D., & Tsoflias, G. P. (2012). An improved vacuum formulation for 2D finite-difference modeling of Rayleigh waves including surface topography and internal discontinuities. Geophysics, 77(1), 1–9.
[6] Zhebel, E., Minisini, S., Kononov, A., & Mulder, W. A. (2014). A comparison of continuous mass-lumped finite elements with finite differences for 3-D wave propagation. Geophysical Prospecting, 63(1), 1111–1125.
[7] Gao, L. et al. (2015) 'An immersed free-surface boundary treatment for seismic wave simulation', Geophysics, 80(5), pp. 193–209.
[8] Louboutin, M. et al. (2019) 'Devito (v3.1.0): an embedded domain-specific language for finite differences and geophysical exploration', Geoscientific Model Development, 12(3), pp. 1165–1187.
[9] Mittal, R., Dong, H., Bozkurttas, M., & Najjar, F. M. (2008). A versatile sharp interface immersed boundary method for incompressible flows with complex boundaries. Journal of Computational Physics, 227, 4825–4852.